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Deterministic Remote Preparation of Electrons States in Coupled Quantum Dots by Stimulated Raman Adiabatic Passage

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Abstract We present a proposal for deterministic remote preparation of electrons states in a semiconductor nanostructure consisting of a single and a double quantum dot. We show that deterministic remote preparation requires a minimum of only one controlled-Not gate plus one Hadamard gate for the basis transformation, and one Hadamard gate and one single-spin rotation for the reconstruction procedure. Picosecond-scale pulses allow for ultra-short total duration of the protocol, which implies a high remote preparation fidelity.

Keywords Remote electrons state preparation · Hadamard gate · Quantum dot

Quantum information science brings us into a whole new era, so that the information can be manipulated and processed with quantum mechanical systems. Using the theory of quantum mechanics in the field of information in the recent years has produced many interesting developments, one of the remarkable exhibitions of the fascination of quantum information science is quantum teleportation [1], which can transmit an unknown state from one location to another without sending a physical copy of the initial state. Unlike teleportation, Lo [2], Pati [3] and Bennett *et al.* [4] have presented an interesting new method to transmit pure known quantum state using a prior shared entanglement and some classical communication when the sender knows completely the transmitted state. This communication protocol is called remote state preparation (RSP). The goal of RSP is the same as that of quantum teleportation. The main difference between RSP and teleportation are in that, (1) in RSP, the sender Alice knows the state that she wants the receiver Bob to prepare, in particular, Alice need not own the state, but only know the information about the state, while in teleportation Alice must own the teleported state, but she need not know the state; (2) in RSP, the required

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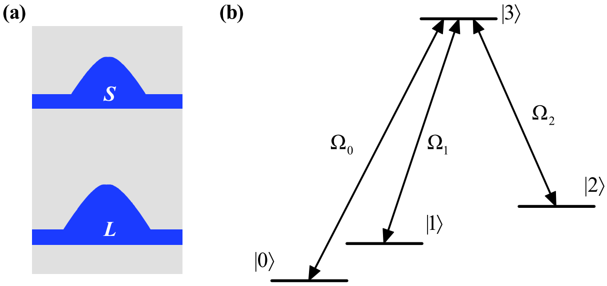

Fig. 1 (**a**) Schematic drawing of vertical section of employed coupled quantum dots. S denotes small dot, L denotes large dot. (**b**) states $|0\rangle$, $|1\rangle$ and $|2\rangle$ are coupled via the state $|3\rangle$ through laser pulses with Rabi frequencies Ω_0, Ω_1, and Ω_2

resource can be traded off between classical communication cost and entanglement cost while in quantum teleportation, two bits of forward classical communication and one ebit of entanglement (an EPR pair) per teleported qubit are both necessary and sufficient, and neither resource can be traded off against the others [4, 5].

So far, RSP has attracted much attention [2–18] in theory, such as low-entanglement RSP [9], higher-dimension RSP [10], optimal RSP [11], RSP for multiparties [12], and continuous variable RSP in phase space, [13] etc. On the other hand, some RSP schemes have been implemented experimentally with the technique of NMR [14] and spontaneous parametric down conversion [15, 16]. To our best knowledge, up to now, electron state have not been remote prepared yet.

In this paper, we propose the remote electronic state preparation (RESP) protocol in quantum dot nanostructure. We will show all the qubit operations in our protocol, including the preparation of the entanglement channel, basis transformation and reconstruction of the original state, are performed through the optical quantum gates proposed by Troiani and Molinari [19]. It is implemented by the technique of the stimulated Raman adiabatic passage (STIRAP) [20, 21], which allows for complete population transfer between two long-lived states. This choice of system is motivated by: (1) the high level of control over the number of electrons confined in quantum dots [22]. (2) recent advances in coherent manipulation of single and entangled pairs of spins in these structures [23], and (3) the relative robustness of the electron spin against decoherence [24]. From Ref. [25] we know that compared to photons and ions, however, the coherence time (T_2) of electron spins in quantum dots is expected to be orders of magnitude shorter and forms the primary limiting factor for coherent quantum communication processes such as RESP.

Now, we demonstrate the whole process of our RESP protocol with the elementary quantum gates presented in Ref. [26]. We consider an array of two quantum-dot molecules a and b in the state $|00\rangle_{ab}$, the sender Alice has bit a, and the receiver Bob has bit b. Each quantum-dot molecule (a and b) consists of two vertically coupled quantum dots inside a field-effect structure, see Fig. 1. The lower dot (labeled as L) is large than the upper one (labeled as S). Just like Ref. [19], the electron eigenstates read

$$|0\rangle = |L\rangle \otimes \left|+\frac{1}{2}\right\rangle, \qquad |1\rangle = |L\rangle \otimes \left|-\frac{1}{2}\right\rangle, \qquad |2\rangle = |S\rangle \otimes \left|+\frac{1}{2}\right\rangle, \tag{1}$$

where $|L\rangle$, $|S\rangle$ denote the electron wavefunction localized in the lower or upper dot, respectively, and $|\pm\frac{1}{2}\rangle$ is the spin eigenstate along the magnetic field. The states $|0\rangle$ and $|1\rangle$ serve us for encoding the qubit, whereas state $|2\rangle$ is an auxiliary state that will be occupied only in the intermediate phase of the rotation procedure. Next, we introduce as a fourth auxiliary state $|3\rangle$, which is a charged-exciton state, consists of two electrons and one hole. Because the interdot tunnelling is suppressed, the electron states $|0\rangle$, $|1\rangle$, and $|2\rangle$ can only be optically coupled via state $|3\rangle$ by different laser fields.

Let us assume that the sender Alice wishes to help the receiver Bob remotely prepare of the state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. First we prepare the entangled pair of bits a and b. We must perform a Hadamard (H) gate on qubit a and one controlled-Not (c-Not) gate between the control bit a and target bit b. As a major improvement, we propose to perform all quantum gates solely by means of sequences of stimulated Raman adiabatic passage (STIRAP) pulses. This technique was originally developed in the field of atomic physics [20, 21]. Recently Kis *et al.* [26] extended this original STIRAP level scheme to an additional long-lived auxiliary state and showed that within the resulting model it becomes possible to perform generic quantum gates. For the sake of clarity, let us briefly rephrase the main steps of this control within the present scheme. It consists of two STIRAP processes: for the first one, two pump pulses with different light polarization selectively address transitions of the system between two long-lived states (here $|0\rangle$ and $|1\rangle$) through optical coupling to an interconnecting state $|3\rangle$. To avoid radiative environment losses of $|3\rangle$, one exploits the renormalized radiation-mater states for the transfer process, which is achieved by slowly varying the exciting laser fields and keeping the population of the state $|3\rangle$ negligible throughout; For the second one, one stokes pulse couples the state $|2\rangle$ and $|3\rangle$. The light polarization of the stokes pulse is the same as the pump pulse couples $|0\rangle$ and $|3\rangle$, but its centre frequency is evidently different. The Rabi frequencies are $\Omega_0(t) = \Omega(t)\cos\theta$, $\Omega_1(t) = \Omega(t)\sin\theta\exp(i\phi)$ and $\Omega_2(t) = \Omega'(t)\exp(i\delta')$, respectively. Where $\Omega(t)$ and $\Omega'(t)$ are the envelopes with different means, and θ and ϕ are fixed angles. The stokes pulse precedes the pump pulses. During this process, the dark state within the qubit subspace would not be affected, but its orthogonal state will be transferred to the state $|2\rangle$. The second process is a reverse of the first one, where the state $|2\rangle$ is mapped back to the bright state accompanied by a phase difference $\delta = |\delta^{'} - \delta^{''}|$. Therefore, after the two STIRAP processes the final state $|\Psi\rangle_f$ may be described as performing a general spin rotation on the initial state $|\Psi\rangle_i$: $|\Psi\rangle_f = e^{-i\delta/2}R_n(\delta)|\Psi\rangle_i$, where $R_n(\delta)$ denotes the spin rotation operator around the axis $n = (\sin 2\theta\cos\phi, \sin 2\theta\sin\phi, \cos 2\theta)$. One can achieve accurate spin rotation by choosing different laser parameters.

Two sequences of STIRAP pulses are applied successively to perform one H gate on a, and one c-Not gate between the control bit a and target bit b. Here we demonstrate the elementary quantum gates employed in our protocol. First, the Hadamard gate:

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{2}$$

which can be translated into one single-spin rotation $H = -iR_n(\pi)$ around $n = (-\sqrt{2}/2, 0, -\sqrt{2}/2)$ With the field parameters $\theta = 3\pi/8$ and $\phi = \pi$, the state after the STIRAP processes is written as

$$|\Psi\rangle_f = e^{-i\pi/2}R_n(\pi)|\Psi\rangle_i = H|\Psi\rangle_i. \tag{3}$$

Second, to perform the controlled-Not gate, two adjacent quantum dot molecules are employed as the control and target qubits (suffix with c and t respectively). First, one STIRAP process with the field parameters $\theta = \pi/2$ and $\phi = 0$ is applied to the control bit. Then, the state $|1\rangle_c$ is transferred to $|2\rangle_c$, the electron makes the transition from the large to the small dot. Then one Not gate implemented by two STIRAP processes is applied to the target bit with the changed transition frequency measured beforehand. the Not gate:

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{4}$$

which can be translated into one rotation $X = iR_x(\pi)$ around the x-axis. With the field parameters $\theta = \pi/4$ and $\phi = 0$, through the STIRAP protocol we obtain

$$|\Psi\rangle_f = -iR_x(\pi)|\Psi\rangle_i = -X|\Psi\rangle_i, \tag{5}$$

where the minus is the global phase. Finally, a reverse STIRAP process is applied to transfer the control bit back to $|1\rangle_c$, it remains through the first STITAP processes the c-Not gate is constructed as

$$\begin{aligned} |00\rangle_{ct} &\to |00\rangle_{ct}, \\ |01\rangle_{ct} &\to |01\rangle_{ct}, \\ |11\rangle_{ct} &\to -|10\rangle_{ct}, \\ |10\rangle_{ct} &\to -|11\rangle_{ct}. \end{aligned} \tag{6}$$

These pulse sequences drive the system $|00\rangle_{ab}$ into the state

$$\begin{aligned} |00\rangle_{ab} &\to \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{ab} \\ &\to \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{ab}, \end{aligned} \tag{7}$$

where the global phase follows (6). Thus, after the H gate and the c-Not gate, bits a and b are in the maximally entangled state and can be used as the quantum channel to realize the RESP.

In order to achieve RESP, Alice has to perform a H gate operation on her bit a by a sequence of STIRAP pulse. Equation (7) can be expanded as

$$|\psi\rangle_{ab} = \frac{1}{2}\{|0\rangle_a(|0\rangle + |1\rangle)_b + |1\rangle_a(|0\rangle - |1\rangle)_b\}. \tag{8}$$

Alice performs a measurement on the bit a. For electron spins in quantum dots, however, no measurement technique is available for full Bell measurements: Current techniques allow one to distinguish singlet from triplets and to perform full measurements in the standard basis. So, the measurement in our protocol is available. Alice informs Bob of his measurement results through a classical information (1 bit). According to the outcome received from Alice, Bob can reconstruct the state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ by applying an appropriate unitary transformation which can be translated into one single-spin rotation (see Table 1), and implemented by a sequence of STIRAP processes to his bit b, with laser parameters $\theta = 0$, $\phi = 0$ and $\delta = \pi$, to rotate the spin around z by π.

In the last section, we discuss the feasibility of the proposed RESP protocol. (1) Our protocol only need c-Not gate operation and H gate operation for the basis transformation. These operations consist of four single-spin rotations in total (c-Not gate requires two rotations and H gate requires one). Moreover, because qubit rotations by the STIRAP protocol bring only global phases, Bob needs only one single-spin rotation to reconstruct the original state. (2) The short total duration of the RESP process. Four single-spin rotation

Table 1 Corresponding relations among Alice's measurement results, Bob's states, and Bob's operations

Alice's measurement results	Bob's state	Unitary transformation	Spin rotation by STIRAP	Final state
$\|0\rangle_a$	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)_b$	–	–	$\|\Psi\rangle$
$\|1\rangle_a$	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)_b$	σ_z	$-iR_x(\pi) = -\sigma_z$	$-\|\Psi\rangle$

for basis transformation and one single-spin rotation for reconstruction procedure are required for the RESP. Every single-spin rotation is implemented by two STIRAP processes, therefor, the protocol needs ten STIRAP processes altogether to remote prepare the state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. From Ref. [27] we know that in experiment, picosecond-scale pulses with duration ~30 ps can be applied to perform coherent rotation of a single electron spin in a quantum dot. The STIRAP protocol used in our protocol requires the pulses partically overlap, so the duration of every STIRAP process is estimated to be double of ~30 ps, leading to ~60 ps. Assuming that single-shot readout is fast, we thus estimated that the total duration of our protocol is $t \sim 600$ ps. On the other hand, because of the effect of environment, mainly the hyperfine interaction between electron and nuclear spins of the dot, electron spin has a decoherence time $T^* \sim 10$ ns [28]. Our protocol is therefore much faster than the spin lifetime, and after only $\sim 6\%$ of T^* the RESP protocol can be completed. Assuming the same interaction with environment, the shorter total duration leads to higher fidelity of the RESP protocol. Thus, it is reasonable to be confident that our protocol has a higher fidelity then others. (3) For electron spins in quantum dots, however, no measurement technique is available for full Bell measurement: Current techniques allow one to distinguish singlet from triplets and to perform full measurements in the standard basis [29]. We therefore use an idea proposed by Brassard *et al.* [30], which consists of measuring the spins in the latter basis (this approach was also used in the NMR and ion experiment).

In summary, we have presented a deterministic remote preparation protocol of electrons state in an array of two double-coupled quantum dots. We demonstrate that our protocol only requires one c-Not gate, two H gate and one single-spin rotation to transfer the state. The quantum gates employed are translated into optical single-spin rotations and implemented by the extended STIRAP protocol. Benefited from the picosecond-scale STIRAP control, the total duration t in our protocol are much smaller than the spin decoherence time T^*, which implies a higher remote preparation fidelity and thus indicates that the reliable large-scale quantum computation with quantum dots is promising. The available experimental techniques including engineering of vertically coupled self-assembled quantum dots [31], picosecond-scale coherent rotation of a single electron spin [27] and single-shot readout of electron spin states [32] provide solid ground for the feasibility of the protocol. We hope that with the existing technology it may be possible to implement the RESP protocol with ease.

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